Kinodynamic Motion Planning for an X4-Flyer by Switching Harmonic Potential Fields

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***Abstract*—In this research, we present a control method using kinodynamic motion planning based on a harmonic potential field (HPF) for an X4-Flyer moving in a 3-dimensional space. In the previous research, it was confirmed that a controller using two HPFs generated on the X-Y and X-Z planes was able to guide the X4-Flyer to the arbitrary target point in a 3-dimensional space. In this paper, the previous method is extended to the case where three HPFs generated on the X-Y, X-Z, and Y-Z planes are used, and it is verified that the X4-Flyer can move efficiently by using the proposed method through some simulations.**

***Index Terms*—Kinodynamics, Motion Planning, Aerial robot.**

I. Introduction

Recently, there are many researches on the autonomous locomotion for an X4-Flyer[1]-[3]. An X4-Flyer is a vertical takeoff and landing (VTOL) aerial robot with four rotors, and it has received attention in recent years as search and rescue robots because of its highly maneuverability and hovering ability. In traditional motion planning, dynamic constraints and kinematic constraints are generally solved separately. In that case, dynamic constraints are mostly solved by designing the control input according to the result of kinematic constraints. There is "kinodynamic motion planning" that is aimed at solving these two constraints simultaneously, for designing the control input from the current state[4]. Kinodynamic motion planning can design the control input in one-step, and therefore it has an advantage of being able to decide the control input simply, compared to existing motion planning. There are many techniques for kinodynamic motion planning, and kinodynamic motion planning based on using "Harmonic potential field (HPF)" was proposed as one of them[5][6].

In this research, a control method is proposed for guiding an X4-Flyer to an arbitrary target point in a 3-dimensional space. The present controller guides an X4-Flyer by appropriately switching the HPF generated in a 2-dimensional plane. It was already confirmed in the previous research that a controller using two HPFs generated on the X-Y and X-Z planes can guide the X4-Flyer to the arbitrary target point in a 3-dimensional space. In this paper, the previous method is extended to the case where three HPFs generated on the X-Y, X-Z, and Y-Z planes are used. Moreover, it is verified by simulations that the X4-Flyer can move to the arbitrary target point efficiently by using the present method based on 2-dimensional HPFs.

II. Kinodynamic Motion Planning for an X4-Flyer

In the proposed method, kinodynamic motion planning is achieved by combining nonholonomic control input and the gradient which is calculated from the HPF. The system input $\boldsymbol{U} = [U_1\ U_2\ U_3\ U_4]^{\mathrm{T}}$, which is constructed by nonholonomic control input $\boldsymbol{u}_{\mathrm{c}}$ and control input $\Delta\boldsymbol{u}$ based on the gradient of the HPF, is as follows:

$$\boldsymbol{U} = \boldsymbol{u}_{\mathrm{c}} + \Delta\boldsymbol{u} \tag{1}$$

Here, U_1 is a control input for acting on each translational motion, and U_2, U_3 and U_4 are control inputs for acting on roll angle ϕ, pitch angle θ and yaw angle ψ, respectively. In the following subsections, we describe the dynamical model of an X4-Flyer, the control input based on nonholonomic control $\boldsymbol{u}_{\mathrm{c}}$ and the proposed control input $\Delta\boldsymbol{u}$ based on the gradient of an HPF.

A. Nonholonomic Control Input

The control input $\boldsymbol{u}_{\mathrm{c}} = [u_{\mathrm{c}1}\ u_{\mathrm{c}2}\ u_{\mathrm{c}3}\ u_{\mathrm{c}4}]^{\mathrm{T}}$ is added for z direction and three attitude angle and given as follows[7]:

$$\begin{aligned} u_{\mathrm{c}1} &= \frac{mg}{\cos\phi\theta} - \frac{m\hat{U}_1}{\cos\phi\theta} \\ u_{\mathrm{c}2} &= -\frac{I_x}{l}(\phi - \phi_{\mathrm{T}}) - k_1\dot{\phi} \\ u_{\mathrm{c}3} &= -\frac{I_y}{l}(\theta - \theta_{\mathrm{T}}) - k_2\dot{\theta} \\ u_{\mathrm{c}4} &= -I_z(\psi - \psi_{\mathrm{T}}) - k_3\dot{\psi} \end{aligned} \tag{2}$$

where $\hat{U}_1$ is

$$\hat{U}_1 = k_4(z - z_{\mathrm{T}}) + k_5\dot{z} \tag{3}$$

In this equation, $k_1, \ldots, k_5$ are positive constant gains, and z_{T} is an arbitrary altitude and $(\phi_{\mathrm{T}}, \theta_{\mathrm{T}}, \psi_{\mathrm{T}})$ are the desired angles.

B. Added Control Input

In this subsection, an added control input $\Delta\boldsymbol{u}$ is described for the translational motion. Kinodynamic motion planning which is proposed in the previous research can only guide an X4-Flyer on the X-Y plane, because it uses an HPF generated in the X-Y plane. The proposed method can control the Z direction of an X4-Flyer by switching an HPF including the Z direction and an HPF in the X-Y plane.

For the control in the X-Y plane, the HPF on the X-Y plane including the current position of the X4-Flyer is used. At that time, using the gradient of the HPF on the X-Y plane, an added control input $\Delta\boldsymbol{u}$ is designed by

$$\begin{cases} \left(\text{if } \sigma < \sqrt{(x-x_{\mathrm{T}})^2+(y-y_{\mathrm{T}})^2}\right) \\ \Delta\boldsymbol{u} = -b_{\mathrm{c}}\dot{\boldsymbol{x}} - k_{\mathrm{v}}\nabla V(\boldsymbol{x}) - k_{\mathrm{c}}F_{\mathrm{C}}(\boldsymbol{x},\dot{\boldsymbol{x}}) \\ (\text{else}) \\ \Delta\boldsymbol{u} = -b_{\mathrm{d}}\boldsymbol{h}(\boldsymbol{x},\dot{\boldsymbol{x}}) - k_{\mathrm{v}}\nabla V(\boldsymbol{x}) \end{cases} \quad (4)$$

$$\boldsymbol{h}(\boldsymbol{x},\dot{\boldsymbol{x}}) = \left[\mathbf{n}^{\mathrm{T}}\dot{\boldsymbol{x}}\mathbf{n} + \left(\frac{\nabla V(\boldsymbol{x})^{\mathrm{T}}}{\|\nabla V(\boldsymbol{x})\|}\cdot\dot{\boldsymbol{x}} \cdot \Phi(\nabla V(\boldsymbol{x})^{\mathrm{T}}\dot{\boldsymbol{x}})\right)\frac{\nabla V(\boldsymbol{x})^{\mathrm{T}}}{\|\nabla V(\boldsymbol{x})\|}\right]$$

$$\boldsymbol{F}_{\mathrm{C}}(\boldsymbol{x},\dot{\boldsymbol{x}}) = (\boldsymbol{x}_{\mathrm{T}}-\boldsymbol{x})\cdot\Phi(\sigma - |\boldsymbol{x}_{\mathrm{T}}-\boldsymbol{x}|)\cdot\Phi(\dot{\boldsymbol{x}}^{\mathrm{T}}(\boldsymbol{x}_{\mathrm{T}}-\boldsymbol{x}))$$

where b_{C}, b_{d}, k_{v} and k_{c} denotes a positive constant gain, $\boldsymbol{h}(\boldsymbol{x},\dot{\boldsymbol{x}}) = [0\ h(y,\dot{y})\ h(x,\dot{x})\ 0]^{\mathrm{T}}$, $\dot{\boldsymbol{x}} = [0\ \dot{y}\ \dot{x}\ 0]^{\mathrm{T}}$, $\nabla V(\boldsymbol{x}) = [0\ f_y\ f_x\ 0]^{\mathrm{T}}$, and $\boldsymbol{F}_{\mathrm{C}}(\boldsymbol{x},\dot{\boldsymbol{x}}) = [0\ F_{\mathrm{C}}(y,\dot{y})\ F_{\mathrm{C}}(x,\dot{x})\ 0]^{\mathrm{T}}$. Note that, f_x and f_y mean the gradient of HPF parallel to the direction of x- and y-axis respectively.

For the Z-directional motion control, the HPF on the X-Z or the Y-Z plane including the current position of the X4-Flyer is used. In this control, the X4-Flyer moves in the Z-direction by subtracting the Z-directional gradient $f_x$calculated with the HPF from the current altitude z and regarding it as the target altitude z_{T} in Eq. (3), so it follows that

$$\hat{U} = k_4(z - (z_{\mathrm{T}} - f_z)) + k_5\dot{z} \quad (5)$$

C. Switching of the HPFs

The proposed method uses 2-dimensional HPFs for guiding the X4-Flyer to an arbitrary target point in a 3-dimensional space. Only two HPFs generated on X-Y and X-Z planes are applied in the previous research, whereas three HPFs are used in this method. Fig. 1 shows a schema of generating the X-, Y-, and Z-directional gradients using three HPFs. In this method, the HPFs are generated on the X-Y, X-Z and Y-Z planes including the current position of the X4-Flyer as shown in Fig. 1. By giving the X-, Y-, and Z-directional gradients to the X4-Flyer, the controller guides the X4-Flyer in the 3-dimentional space. Here, the variables f_x, f_y, and f_z in Fig. 1 are X-, Y-, and Z-directional gradients, respectively.

As an illustration, consider the case where the X4-Flyer moves from the current position $(x_0, y_0, z_0) = (45, 25, 10)$ to the target position $(x_{\mathrm{T}}, y_{\mathrm{T}}, z_{\mathrm{T}}) = (5, 5, 10)$ in the environment shown in Fig. 2. In this case, the HPF on the X-Y plane including the current altitude z = 1 (see Fig. 3 (a)), the HPF on the X-Z plane including the current y-position y = 25 (see Fig. 4 (a)), and the HPF on the Y-Z plane including the current x-position x = 45 (see Fig. 5 (a)) are generated. In order to generate the HPFs, $(x_{\mathrm{T}}, y_{\mathrm{T}}) = (5, 5)$ i is set as a target point in Fig. 3, $(x_{\mathrm{T}}, z_{\mathrm{T}}) = (5, 10)$ i is set as a target point in Fig. 4, and $(y_{\mathrm{T}}, z_{\mathrm{T}}) = (5, 10)$ is set as a target point in Fig. 5. Then, by using the gradient of the current position generated as in Fig. 3 (b), Fig. 4 (b), and Fig. 5 (b), the controller guides the X4-Flyer toward the target point

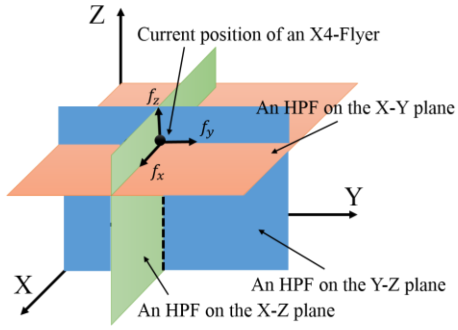


Fig. 1 A schema of generating 3-dimentional gradient using two HPFs

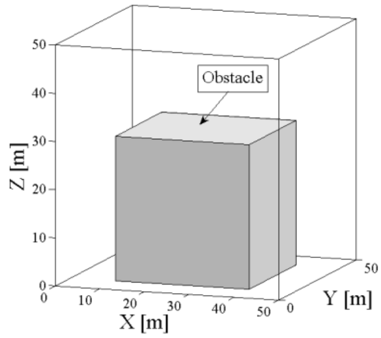
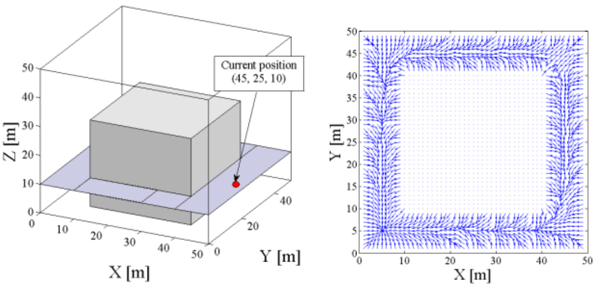
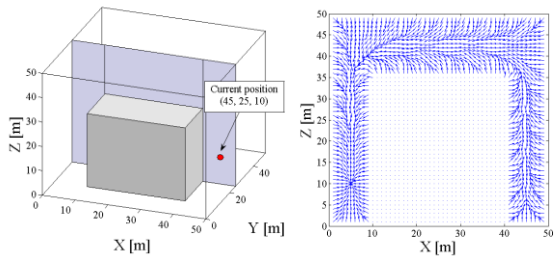


Fig. 2 The assumed field



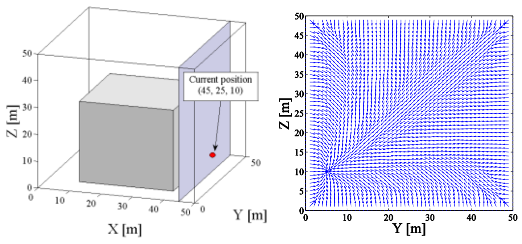
(a) X-Y plane including the current position (b) The gradient of the HPF on the X-Y plane

Fig. 3 The HPF on the X-Y plane



(a) X-Z plane including the current position (b) The gradient of the HPF on the X-Z plane

Fig. 4 The HPF on the X-Z plane



(a) Y-Z plane including the current position (b) The gradient of the HPF on the Y-Z plane

Fig. 5 The HPF on the Y-Z plane

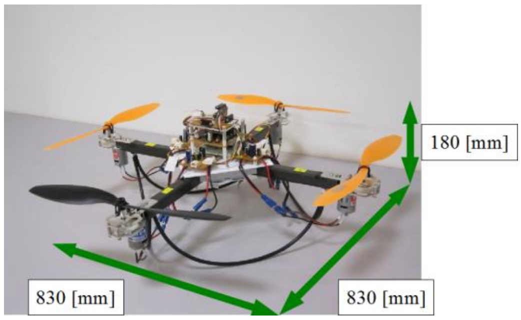


Fig. 6 TheX4-Flyer

III. SIMULATIONS

In this section, it is confirmed that the proposed method can guide the X4-Flyer 3-dimensionally in the environment shown in Fig. 2, by simulations in MATLAB. Additionally, the proposed method is compared with the previous method to verify that the proposed method is more efficient than the previous one.

A. Conditions

In this simulation, it is assumed that the X4-Flyer moves from the initial position $(x_0, y_0, z_0) = (45, 25, 10)$ to the target position $(x_T, y_T, z_T) = (5, 5, 10)$ in the environment including an obstacle (shown in Fig. 2). The target attitude is always set as $(\phi_T, \theta_T, \psi_T) = (0, 0, 0)$, and the following equation is used as a dynamical model of the X4-Flyer[7]:

$$\begin{aligned}
\ddot{x} &= -(\cos\phi\sin\theta\cos\psi + \sin\phi\sin\psi)\frac{1}{m}U_1 \\
\ddot{y} &= -(\cos\phi\sin\theta\sin\psi - \sin\phi\cos\psi)\frac{1}{m}U_1 \\
\ddot{z} &= -g + (\cos\phi\cos\theta)\frac{1}{m}U_1 \\
\ddot{\phi} &= \dot{\theta}\dot{\psi}\left(\frac{I_y - I_z}{I_x}\right) - \frac{J_r}{I_x}\dot{\theta}\Omega + \frac{l}{I_x}U_2 \\
\ddot{\theta} &= \dot{\phi}\dot{\psi}\left(\frac{I_z - I_x}{I_y}\right) - \frac{J_r}{I_y}\dot{\phi}\Omega + \frac{l}{I_y}U_3 \\
\ddot{\psi} &= \dot{\phi}\dot{\theta}\left(\frac{I_x - I_y}{I_z}\right) + \frac{1}{I_z}U_4
\end{aligned} \tag{6}$$

In this equation, let us define m [kg] as the mass of the X4-Flyer, l [m] as the length from the center of airframe to the center of rotor, g [m/s2] as the gravity acceleration, I_x, I_y and I_z [kg/m2] as the inertial moment around each axis respectively, and J_r [kg/m^2] as the inertial moment of a rotor. Here, U_1 is the control input for acting on each translational motion, and U_2, U_3 and U_4 are the control inputs for acting on roll motion, pitch motion and yaw motion respectively. Then, Ω and the system's inputs U_1, U_2, U_3, U_4 can be written by using the rotational speed $\omega_i$of the rotor i (i=1,...,4), i.e

Table 1 Model parameters for the X4-Flyer

Parameter	Description	Value	Unit
$\boldsymbol{g}$	Gravity	9.80665	$\mathrm{m/s^2}$
$\boldsymbol{m}$	Mass	1.3	kg
$\boldsymbol{l}$	Distance	0.248	m
$\boldsymbol{I_x}$	Roll inertia	0.01467	$\mathrm{kg \cdot m^2}$
$\boldsymbol{I_y}$	Pitch inertia	0.01467	$\mathrm{kg \cdot m^2}$
$\boldsymbol{I_z}$	Yaw inertia	0.02331	$\mathrm{kg \cdot m^2}$
$\boldsymbol{J_r}$	Rotor inertia	175.69×10^{-6}	$\mathrm{kg \cdot m^2}$
$\boldsymbol{b}$	Thrust factor	0.0000434	
$\boldsymbol{d}$	Drag factor	0.000002188	

Table 2 Controller gains for the X4-Flyer

Gain	Value	Gain	Value
$\boldsymbol{k_1}$	0.015	k_2	0.015
$\boldsymbol{k_3}$	0.007	k_4	10.0
$\boldsymbol{k_5}$	25.0	k_v	0.001
$\boldsymbol{k_c}$	0.002	b_d	0.002
$\boldsymbol{b_c}$	0.004	σ	10

$$\begin{aligned}
U_1 &= b(\omega_1^2 + \omega_2^2 + \omega_3^2 + \omega_4^2) \\
U_2 &= b(\omega_4^2 - \omega_2^2) \\
U_3 &= b(\omega_3^2 - \omega_1^2) \\
U_4 &= d(\omega_2^2 + \omega_4^2 - \omega_1^2 - \omega_3^2) \\
\Omega &= \omega_2 + \omega_4 - \omega_1 - \omega_3
\end{aligned} \tag{7}$$

Here b is the thrust coefficient and d is the drag coefficient. The parameters used for this simulation are shown in Table 1, and they are obtained from the X4-Flyer developed in our laboratory (shown in Fig. 6). The gains are set as in Table II, from a rule of thumb.

B. Results

Figs. 7-10 show the simulation results. The solid red line drawn in Fig. 7 denotes the trajectory of the X4-Flyer when using the proposed method, and the dotted blue line denotes the trajectory when using the previous method. Fig. 8 shows the change of the Euclidean distance between the current position and the target position, and Fig. 9 and Fig. 10 show the change of the attitude angles (ϕ, θ, ψ) when using the proposed method and the previous method respectively.

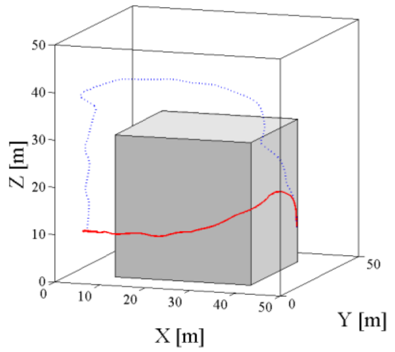


Fig. 7 The trajectory of the X4-Flyer

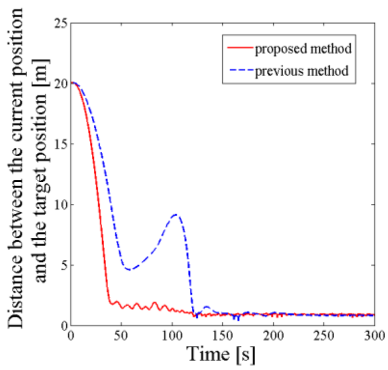


Fig. 8 Distance from the target position

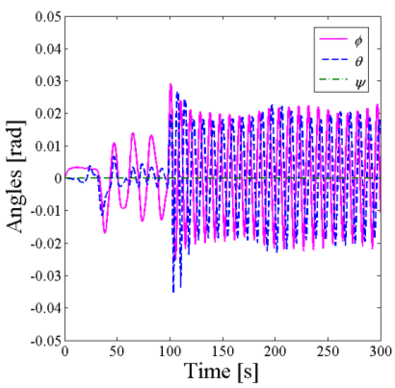


Fig. 9 Angle of the X4-Flyer using the proposed method

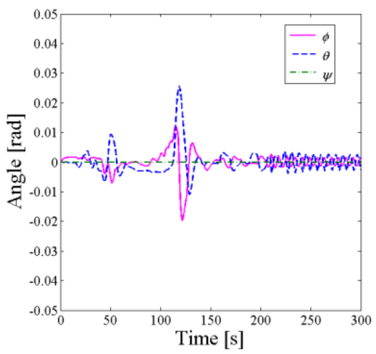


Fig. 10 Angle of the X4-Flyer using the previous method

C. Discussions

It is found from Fig. 7 that the X4-Flyer was able to reach the target point from the initial state while avoiding the obstacle in the 3-dimensional environment by using the proposed method or the previous method. Moreover, the X4-Flyer was able to move 3-dimensionally by using the gradient of HPFs in spite of only using the 2-dimensional HPFs. However, the trajectory when using the previous method takes a long detour to upside of the obstacle. On the other hand, the trajectory when using the proposed method takes a shorter route from the side of the obstacle and it seems that an efficient trajectory was chosen.

As shown in Fig. 8, the X4-Flyer was able to keep its attitude on the target point after reaching the target point even if which controller was used. However, the proposed method was able to guide the X4-Flyer to the target point about two times faster than the previous one. Small steady-state errors remained after the X4-Flyer reached the target point, irrespective of the control method adopted.

All the attitude angles of the X4-Flyer always fell within $\pm$ 0.04 [rad] as shown in Figs. 9 and 10. This result is acceptable realistically, because the X4-Flyer assumed in this simulation was able to easily cope with the oscillation of about $\pm$ 0.2 [rad]. From these results, it is confirmed that the proposed method can choose a more efficient route and guide the X4-Flyer to the target point faster than the previous method.

IV. Conclusions

In this paper, a method for guiding the X4-Flyer to an arbitrary target point in a 3-dimensional space is proposed by switching the HPFs, which are generated on 2-dimensional planes. The usability of the proposed method was confirmed in simulations by comparing it with the previous method. In future works, we are going to implement the proposed method to an actual machine, and make experiments in actual environments.

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