Simulation and Experimental Works of Quadcopter Model for Simple Maneuver

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Abstract—**This study aims to create a simulated and experimental of aircraft movements for multirotor quadcopter. The research method is theoretical and experimental methods. For theoretical method consists of calculating the dynamics and kinematics. While the experimental method consists of the aircraft testing and processing of GPS data recorded aircraft. The results showed that the acceleration acting on the aircraft is large enough that x¨ = 1.751 m/s2, y = 2.038 m /s2 = 1.6371 m danz¨/ s2, (2) the value of the maximum error between the theoretical and the actual movement is ex = 0.682 m; ey and ez = 0.353 m = 0.546 m. Theoretical movement pattern already resembles the actual movement..**

Keywords: model simulation, quadcopter, dynamic control, kinematics control

I. Introduction

One type robots attract much attention is a mini unmanned aerial aircraft UMAVs ("*Unmanned Mini Aerial Vehicles*"), because of its ability to perform rescue tasks in hazardous locations and difficult to reach. This type of helicopter flying robot has the advantage over other flying vehicle that can maneuver in cramped areas and perform takeoff and vertical landing that it is called *vertical take-off landing* (VTOL).

Figure 1. Quadcopter with 4 rotors

Adapun penelitian ini akan membahas suatu jenis pesawat mini udara tanpa awak dengan tipe sayap berputar RUMAV ("*Rotary-wing Unmanned Mini Aerial Vehicle*") yang dinamakan *quadcopter*. Shown at Figure 1, the quadcopter which is a flying robot that has four blades - independent rotor propeller mounted at each end of a cross frame.

II. Dynamics Model

In general, a quadcopter described simply as four rotors that are in a cross configuration. Vertical movement is obtained by adding or reducing the speed of the rotor with all the same value. This movement produces a vertical force U_1 (N) to the body frame which will be up or down the quadcopter. Roll motion is obtained by increasing or reducing the rotor speed of the left and at the same time reducing or increase the right rotor speed. Pitch motion is obtained in the same way on both the other motors.

Front and rear motors turn to the the anti-clockwise direction while the two other motor turn to the clockwise, so that the direction of the yaw motion/ anti-clockwise obtained if the front-rear speed propeller - increase/ decrease and the left and right speed of propeller decrease/ increase.

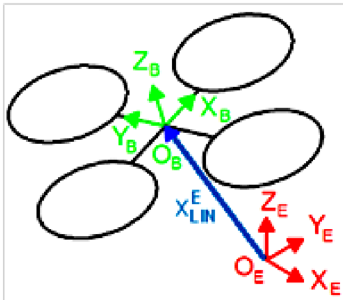

Figure 2. Framework configuration of Quadcopter

To design a dynamic quadcopter model, first, defined two frames (coordinate axes) as shown in Figure 2, condition of an aircraft that explains the position and a vector velocity defined in a state of X_H. Each vector consists of linear and angular position and linear velocity and angle.

If X^E_{LIN} is linear position vector consisting of the components in the direction of the x,y,z (x,y,z) and X^E_{ANG} angular position vector consisting of the components of the angular position x,y,z (ϕ,θ,ψ) then Quadcopter position vector X_H [1] is

$$X_H = [X^E_{LIN} X^E_{ANG}]^T = [x \quad y \quad z \quad \phi \quad \theta \quad \psi]^T \qquad (1)$$

If $\dot{X}^E_{LIN}$ is the linear velocity vector consisting of components of velocity of the x,y,z $(\dot{x},\dot{y},\dot{z})$ and $\dot{X}^B_{ANG}$ is the angular velocity vector consisting of the components of the angular velocity on the x, y, z (p, q, r) then velocity vector,

$$\dot{X}_H = [\dot{X}^E_{LIN}\dot{X}^B_{ANG}]^T = [\dot{x}\dot{y}\dot{z} \quad p \quad q \quad r]^T \qquad (2)$$

Subscript E, B and H indicates that the variable relative to the frame-E, B-frame and H-frame.

It will be distinguished initial state vector and the state vector after acceleration in a given time t, respectively notated X_{H0}, $\dot{X}_{H0}$, X_{Ht} dan $\dot{X}_{Ht0}$,

$$X_{H0} = [x_0 \quad y_0 \quad z_0 \quad \phi_0 \quad \theta_0 \quad \psi_0]^T \qquad (3)$$

$$\dot{X}_{H0} = [\dot{x}_0\dot{y}_0\dot{z}_0 \quad p_0 \quad q_0 \quad r_0]^T \qquad (4)$$

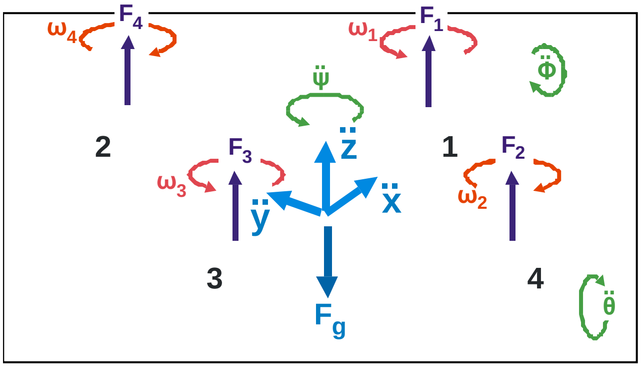

Figure 3. Freebody diagram of quadcopter with four rotors.

Figure 3, shows the dynamics of the movement of a quadcopter. If the aircraft is given an acceleration of the aircraft will change its position and velocity. If $\ddot{X}$ is acceleration vector consisting of components $(\ddot{x}, \ddot{y}, \ddot{z}, \dot{p}, \dot{q}, \dot{r})$, the new velocity vector at time *t* is obtained by the acceleration vector mengintegral quadcopter against *t*. While the new position vector at time *t* is obtained by integration of the velocity vector respect to time *t*. Percepatan yang bekerja pada pesawat diakibatkan oleh gaya yang diberikan pada pesawat.

Then, the equilibrium of the dynamics model of quadcopter will be shown in this part. Then it is important for analyzing the dynamic equilibrium in the plane so it will know the forces and moments that can work on the plane. Furthermore, of the forces and moments acting on the aircraft will be known acceleration occurs.

If the moment of inertia matrix M_H, $\ddot{X}_H$ acceleration matrix $\dot{X}_H$, Sentripetal-Coriolis matrix CH, velocities matrix $\dot{X}_H$, GB gravitational vector and the action vector Λ of the general movement of an aircraft dynamics model of quadcopter can define in the following matrix form [1],[2]:

$$M_H.\ddot{X}_H + C_H.\dot{X}_H - G_H = \Lambda \qquad (5)$$

If the action of the movement vector UH, OH propeller gyroscopic matrix and propeller angular velocity Ω, then the action vector is a general movement

$$\Lambda = U_H + O_H.\Omega \qquad (6)$$

Both equation above can be written in the following form

$$M_H.\ddot{X}_H = -C_H.\dot{X}_H + G_H + O_H.\Omega + U_H \qquad (7)$$

Subscript H indicates that the variable corresponding relative to the H frame.

If m [kg] is mass of *quadcopter* and I [N m s^2] is moment inertia matrix, the the inertia system could be described as,

$$M_H = \begin{bmatrix} mI_{3x3} & O_{3x3} \\ O_{3x3} & I \end{bmatrix} = \begin{bmatrix} m & 0 & 0 & 0 & 0 & 0 \\ 0 & m & 0 & 0 & 0 & 0 \\ 0 & 0 & m & 0 & 0 & 0 \\ 0 & 0 & 0 & I_{xx} & 0 & 0 \\ 0 & 0 & 0 & 0 & I_{yy} & 0 \\ 0 & 0 & 0 & 0 & 0 & I_{zz} \end{bmatrix} \qquad (8)$$

It appears that MH is a diagonal matrix and a is a constant. Centripetal-Coriolis matrix,

$$C_H = \begin{bmatrix} O_{3x3} & O_{3x3} \\ O_{3x3} & -S(I.\dot{X}^B_{ANG}) \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & I_{zz}\,r & -I_{yy}\,q \\ 0 & 0 & 0 & -I_{zz}\,r & I_{yy} & I_{xx}\,p \\ 0 & 0 & 0 & I_{yy}\,q & -I_{xx}\,p & 0 \end{bmatrix} \qquad (9)$$

Gravitational vector,

$$G_H = \begin{bmatrix} F^E_G \\ O_{3x1} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ -mg \\ 0 \\ 0 \\ 0 \end{bmatrix} \qquad (10)$$

If JTP [N m s2] is the total rotational moment of inertia of the propeller axis and Ω [rad / s] is the angular velocity of the propeller whole, then the propeller gyroscopic matrix,

$$O_H\Omega = J_{TP} \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ q & -q & q & -q \\ -p & p & -p & p \\ 0 & 0 & 0 & 0 \end{bmatrix} \Omega \qquad (11)$$

If lift force factor *b* [N s2] and d [N m s2] is the drag factor, then the movement matrix can be shown below,

$$E_H = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ b & b & b & b \\ 0 & -b\,l & 0 & b\,l \\ -b\,l & 0 & b\,l & 0 \\ -d & d & -d & d \end{bmatrix} \qquad (12)$$

If U1 [N], U2 [N m], U3 [N m] and U4 [N m] are the lift force, roll torque, torque pitch and yaw torque respectively, then lift factor b [N s2] and the drag factor d [N m s2] then action vector relative to frame-B,

$$U_B = E_B\,\Omega^2 = \begin{bmatrix} 0 \\ 0 \\ U_1 \\ U_2 \\ U_3 \\ U_4 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ b\,(\Omega_1^2 + \Omega_2^2 + \Omega_3^2 + \Omega_4^2) \\ b\,l\,(\Omega_4^2 - \Omega_2^2) \\ b\,l\,(\Omega_3^2 - \Omega_1^2) \\ d\left(\Omega_2^2 + \Omega_4^2 - \Omega_1^2 - \Omega_3^2\right) \end{bmatrix} \quad (13)$$

If R_Θ rotation matrix then relative action vector aksi relatif respect to frame-H,

$$U_H = \begin{bmatrix} R_\Theta & O_{3x3} \\ O_{3x3} & I_{3x3} \end{bmatrix} U_B = \begin{bmatrix} (s_\psi s_\phi + c_\psi s_\theta c_\phi)U_1 \\ (-c_\psi s_\phi + s_\psi s_\theta c_\phi)U_1 \\ (c_\theta c_\phi)U_1 \\ U_2 \\ U_3 \\ U_4 \end{bmatrix} \quad (14)$$

If c_k = cos k, s_k = sin k then matriks matrix,

$$R_\theta = \begin{bmatrix} c_\psi c_\theta & -s_\psi c_\phi + c_\psi s_\theta s_\phi & s_\psi s_\phi + c_\psi s_\theta c_\phi \\ s_\psi c_\theta & c_\psi c_\phi + s_\psi s_\theta s_\phi & -c_\psi s_\phi + s_\psi s_\theta c_\phi \\ -s_\theta & c_\theta s_\phi & c_\theta c_\phi \end{bmatrix} \quad (15)$$

If t_k = tan k then transfer matrix,

$$T_\theta = \begin{bmatrix} 1 & s_\phi t_\theta & c_\phi t_\theta \\ 0 & c_\phi & -s_\phi \\ 0 & s_\phi/c_\theta & c_\phi/c_\theta \end{bmatrix} \quad (16)$$

Acceleration vector of *quadcopter* relative to frame-B could be described as,

$$\ddot{X}_H = [\ddot{x}\;\ddot{y}\;\ddot{z}\;\dot{p}\;\dot{q}\;\dot{r}] = (-C_H\dot{X}_H + G_B + O_H\,\Omega + U_H)\;M_H^{-1} \quad (17)$$

In the simple equation is described as (17):

$$\begin{aligned} \ddot{x} &= (\sin\psi\sin\phi + \cos\psi\sin\theta\cos\phi)\frac{U_1}{m} \\ \ddot{y} &= (-\cos\psi\sin\phi + \sin\psi\sin\theta\cos\phi)\frac{U_1}{m} \\ \ddot{z} &= -g + (\cos\theta\cos\phi)\frac{U_1}{m} \\ \dot{p} &= \frac{I_{yy} - I_{zz}}{I_{xx}}\,q\,r - \frac{J_{TP}}{I_{xx}}\,q\,\Omega + \frac{U_2}{I_{xx}} \\ \dot{q} &= \frac{I_{zz} - I_{xx}}{I_{yy}}\,p\,r - \frac{J_{TP}}{I_{yy}}\,q\,\Omega + \frac{U_3}{I_{yy}} \\ \dot{r} &= \frac{I_{xx} - I_{yy}}{I_{zz}}\,p\,q + \frac{U_4}{I_{zz}} \end{aligned} \quad (18)$$

On equation (18) shows the mathematical model of quadcopter.

III. DYNAMICS MODEL OF QUADCOPTER

This section will be calculated dynamics of the movement of aircraft with specifications as shown in Table 1 The calculation process will be assisted with Matlab program.

Tabel 1.Spesifikasi Pesawat

Para-meters	Deskripsi	Nilai
L	arm	240 mm
b	lift coefficient	72,081.10^{-6}Ns²
d	drag coefficient	3,433.10^{-6}Nms²
m	mass of quadcopter	1,576 kg
I_{xx}	body inertia moment for axis-x	15,293.10^{-3}Nms²
I_{yy}	body inertia moment for axis-y	15,293.10^{-3}Nms²
I_{zz}	body inertia moment for axis-z	20,307. 10^{-3}Nms²
J_{TP}	rotational moment inertia	0,103. 10^{-6}Nms²

An example of output program using matlab program:

```
                                  |    0.1243|
                                  |    0.1397|
                                  |   26.0946|
Vektor Aksi Pergerakan, U_H =|   -0.0484|
                                  |    0.0003|
                                  |   -0.0002|

Inertia Matrix of the System , M_H,

      |  1.5760   0.0000   0.0000   0.0000  0.0000   0.0000|
      |  0.0000   1.5760   0.0000   0.0000  0.0000   0.0000|
      |  0.0000   0.0000   1.5760   0.0000  0.0000   0.0000|
=     |  0.0000   0.0000   0.0000   0.0153  0.0000   0.0000|
      |  0.0000   0.0000  0.0000   0.0000  0.0001  0.0000  |
      |  0.0000   0.0000   0.0000   0.0000  0.0000  0.0317 |

Sentripetal-Coriolis Matrix C_H (.10^-4)

      | 0.0000  0.0000  0.0000   0.0000    0.0000     0.0000|
      | 0.0000  0.0000  0.0000   0.0000    0.0000     0.0000|
      | 0.0000  0.0000  0.0000   0.0000    0.0000     0.0000|
=     | 0.0000  0.0000  0.0000  -0.0000   -0.1189   32.7469 |
      | 0.0000  0.0000  0.0000   0.1189   -0.0000  -32.7469 |
      | 0.0000  0.0000  0.0000   0.2275  32.7469    -0.0000|

                              |   0.0000     |
                              |   0.0000     |
                              |  -15.4448    |
Gravitasional Vector G_H =|   0.0000     |
                              |   0.0000     |
                              |   0.0000     |

Gyroscopic Propeller Matrix O_H (.10^-4)

      | 0.0000   0.0000   0.0000   0.0000|
      | 0.0000   0.0000   0.0000   0.0000|
      | 0.0000   0.0000   0.0000   0.0000|
=     | 0.1965 -0.1965   0.1965  -0.1965 |
      | 0.2208 -0.2208   0.2208  -0.2208 |
      | 0.0000 -0.0000   0.0000  -0.0000 |

Angular velocities vector of each propeller,

                 |  300.8505|
                 |  302.9173|
Omega      =  |  300.8799|
                 |  298.7114|

Angular velocities of quadcopter, Ω  =  -0.1017
```

Acceleration Vector for aircraft,

$$XDDot\_H = \begin{vmatrix} 0.0789 \\ 0.0886 \\ 6.7575 \\ -3.1632 \\ 2.8624 \\ -0.0252 \end{vmatrix}$$

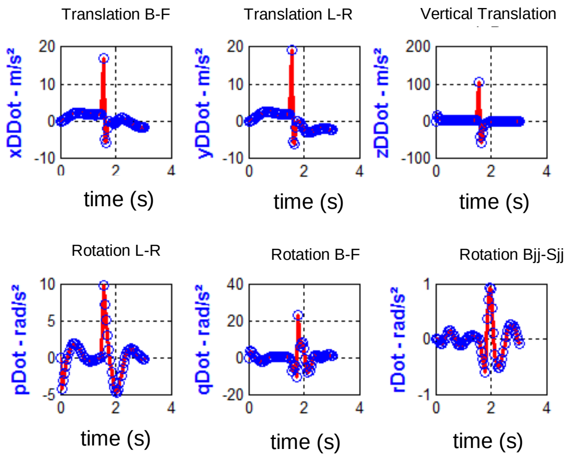

Figure 4. Acceleration and Time for vertical and horizontal movements

Figure 4 shows the acceleration (m/s^2) and angular accelration (rad/s^2) respect to time.

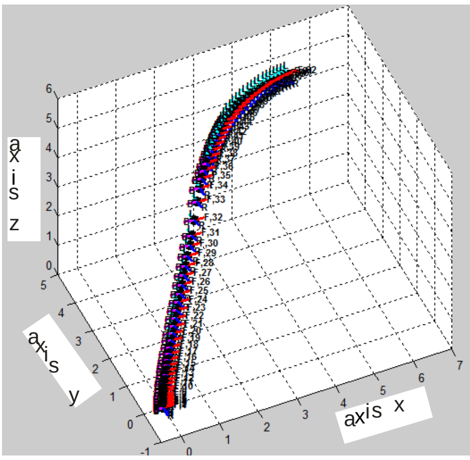

Figure 5.Simulation of aircraft movement

IV. Quadcopter Simulation

This section will show a simulation of movement quadcopter both theoretically and experimentally. Movement of the experimental results obtained from the GPS data attached to Quadcopter. Furthermore, the results of calculations using the model dynamics and kinematics will be demonstrated in this section. For illustration, the movement in the x direction is planned to reach a distance of 5 m. As for the motion in the y direction is planned to reach a distance of 4.5 m. For motion in the z direction is planned and maintained at a height of 5 m.

In Figure 5, shows the graph of the trajectory of aircraft movement in three dimensions obtained from the calculation of dynamics and kinematics using matlab program

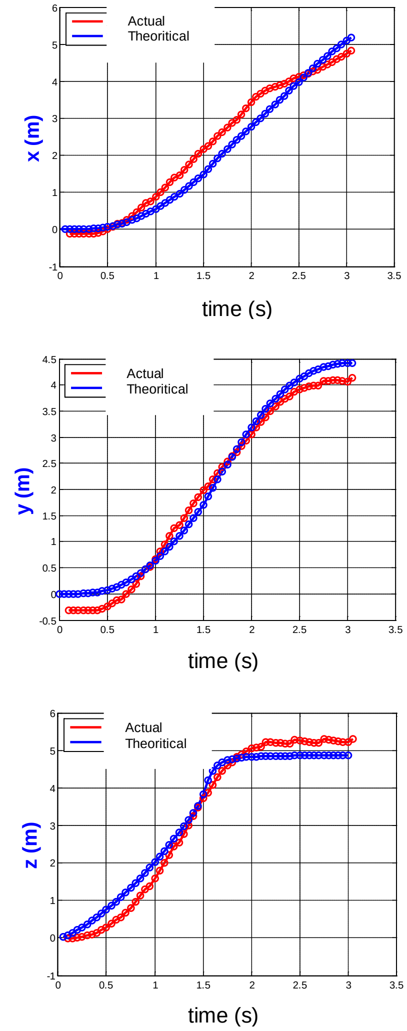

Figure 6. Position (m) vs time t(s)

In Figure 6, shows the graph plane distance (m) and time (s) in the x, y and z both theoretical and experiments using matlab program.

V. Conclusions

Acceleration due to the thrust plane is large enough that $\ddot{x}$ = 1.7509 m / s2, $\ddot{y}$ = 2.0377 m / s2 and $\ddot{z}$ = 1.6371 m / s2 for each mileage is x = 0.5565 m , y = 0.6307 m and z = 1.9700 m. Thus indicating that the aircraft is quite agile in maneuvering.

Quadcopter movement simulation shown that the maximum error position between the theoretical and the actual movement is ex = 0.683 m; ey and ez = 0.353 m = 0.546 m. Movement patterns and the theoretical value of the error is influenced by the control limit membership function of the fuzzy logic control design. Theoretical movement pattern similar to the actual movement.

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